Intrinsic Flat and Gromov-Hausdorff Convergence

Christina Sormani

CUNY GC and Lehman College

Lecture I

Intrinsic Flat and Gromov-Hausdorff Convergence

Lecture 1: Geometric Notions of Convergence

Smooth (C^k), Lipschitz (Lip), Gromov-Hausdorff (GH), Sormani-Wenger Intrinsic Flat (SWIF) Convergence

[Gromov Structures-Metriques] [Sormani-Wenger JDG-2011]

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Almost Rigidity of the Positive Mass Theorem
Geometric Stability of the Scalar Torus Rigidity Theorem
Scalar Sphere Rigidity Theorem and more....
[Conjectures on Conv and Scalar (2021) arXiv: 2103.10093]

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Lectures 3&4: Techniques to Apply to Prove Convergence
Ambrosio-Kirchheim Theory of Integral Currents
Decomposition into Regions with Lakzian
Properties with Portegies and Arzela-Ascoli Theorems
Volume Above Distance Below with Allen and Perales
See https://sites.google.com/site/intrinsicflatconvergence/

Lecture 1: Geometric Notions of Convergence

Goal Today: Build Geometric Intuition

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Viewing Riemannian Manifolds as Metric Spaces

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Recommended Resources:
[Gromov Structures-Metriques]
[Burago-Burago-Ivanov Text]
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A Riemannian Manifold is a Smooth Metric Space

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$$d_g(p, q) = \inf\{L_g(C) : \; C(0) = p, \; C(1) = q\}$$

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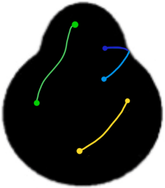
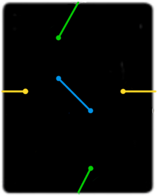
When M is compact, $\exists$ a geodesic, $\gamma_{p,q}$ s.t. $L(\gamma_{p,q}) = d(p,q)$.

A sphere with a bump:



A torus:



C^k Limits of Sequences of Riemannian Manifolds

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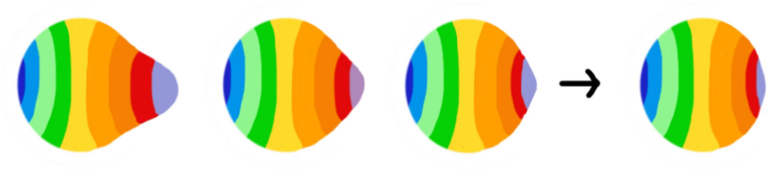
A C^k smooth limit (M_∞, g_∞) is diffeomorphic to the sequence with diffeomorphisms $\psi_j : M_\infty \to M_j$ s.t. $\psi_j^* g_j \to g_\infty$ C^k smoothly on M_∞.



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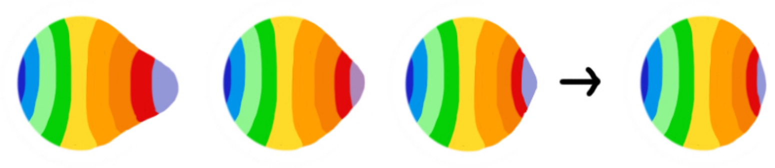
$$\text{Thus:} \qquad d_\infty(x,y) = \lim_{j\to\infty} d_j(\psi_j(x), \psi_j(y)).$$

$$\text{and } \mathrm{Vol}(B_p(R)) = \lim_{j\to\infty} \mathrm{Vol}(B_{\psi_j(p)}(R))$$



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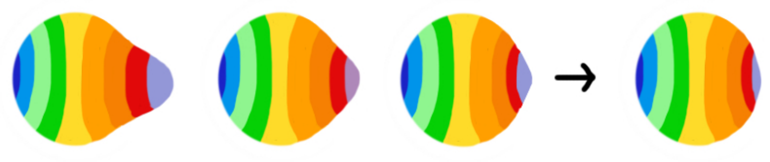

In fact C^0 Convergence $\implies$ Gromov Lipschitz (Lip) Convergence:

$$d_{Lip}((M_j, d_j), (M_\infty, d_\infty)) = Log(\max\{dil(\psi_j), dil(\psi_j^{-1}\}) \to 0$$

where $dil(\psi_j) = \sup\{d_j(\psi_j(x), \psi_j(y))/d_\infty(x,y) \, : \, x \neq y\}$

which is well defined for biLipschitz sequences of metric spaces.

Lip Limits of Sequences of Metric Spaces

A Gromov-Lipschitz limit (M_∞, d_∞) of (M_j, d_j) has biLipschitz maps $\psi_j : M_\infty \to M_j$ s.t. $dil(\psi_j) \to 1$ and $dil(\psi_j^{-1}) \to 1$.

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Examples with no C^k or *Lip* limit:

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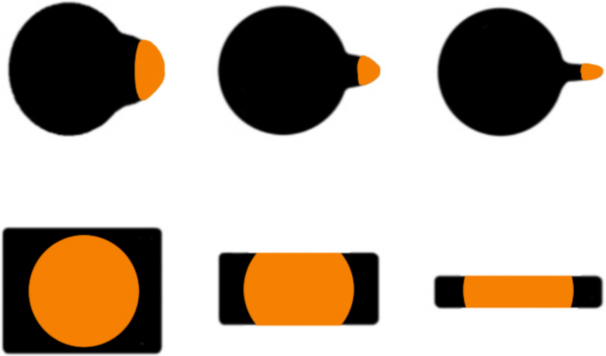

Gromov Hausdorff Limits via Almost Isometries

Gromov's Defn: Compact metric spaces $(X_j, d_j) \xrightarrow{\text{GH}} (X_\infty, d_\infty)$

iff $\exists$ ε_j-*almost isometries* $\psi_j : X_\infty \to X_j$ where $\varepsilon_j \to 0$.

This means that ψ_j are ε_j-*almost distance preserving:*

$$|d_\infty(x,y) - d_j(\psi_j(x), \psi_j(y))| < \varepsilon_j \quad \forall x, y \in X_\infty$$

and ε_j-*almost onto*: $\quad X_j \subset T_{\varepsilon_j}(\psi_j(X_\infty))$.

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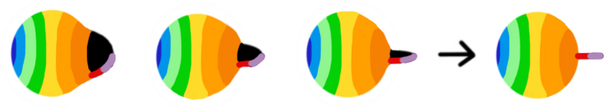

Here our rainbow drawn ψ_j are not onto nor continuous:

Gromov Hausdorff Limits via Nets

Gromov's Prop: Compact $(X_j, d_j) \xrightarrow{\text{GH}} (X_\infty, d_\infty)$ iff

$\exists\ \varepsilon_j \to 0$ and there exist ε_j-nets $S_j \subset X_j \subset T_{\varepsilon_j}(S_j)$ which are finite

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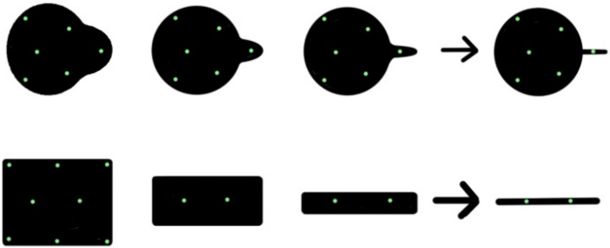

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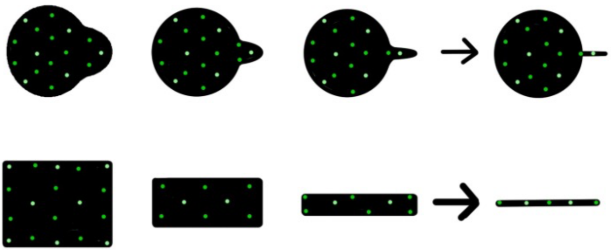

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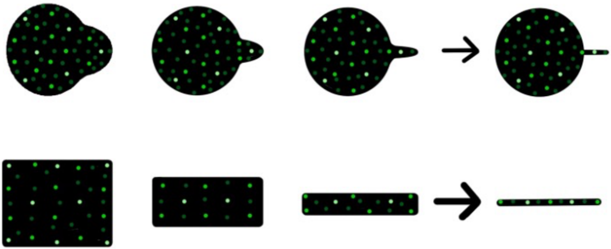
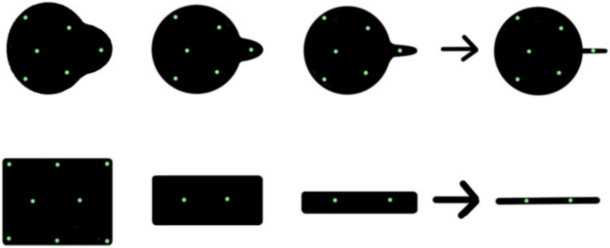


Gromov's Compactness Theorem

If (X_j, d_j) are compact metric spaces with $\mathrm{Diam}(X_j) \leq D$ such that $\forall r > 0\ \exists$ r-nets S_j^r with cardinality $N(r)$ not depending on j then a subsequence $(X_{j_k}, d_{j_k}) \xrightarrow{\mathrm{GH}} (X_\infty, d_\infty)$ where X_∞ is compact.



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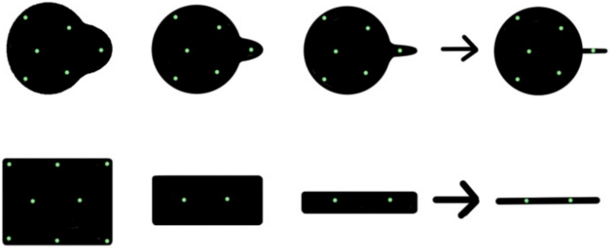

Pf: Fix $r > 0$: d_j restricted to $S_j^r \times S_j^r$ is an $N(r) \times N(r)$ matrix.

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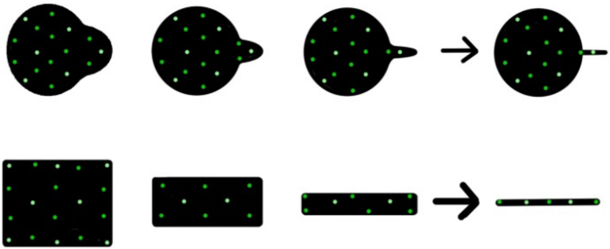

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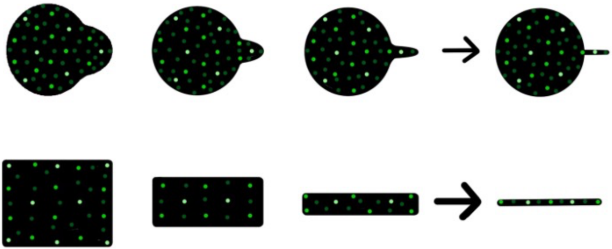

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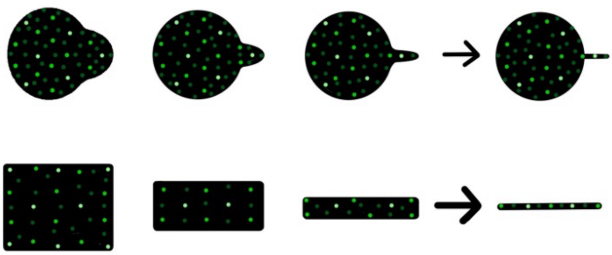

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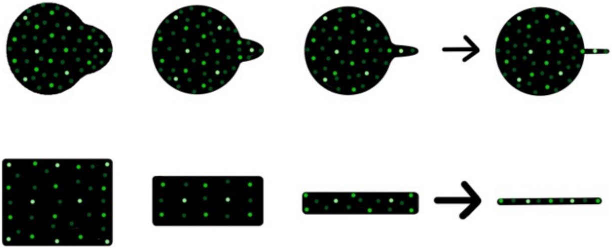

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Gromov Thm: $\mathrm{Diam}(M_j) \le D$ and $Ricci_j \ge H \implies \exists M_{j_k} \xrightarrow{\mathrm{GH}} X_\infty$

GH as Intrinsic Hausdorff Distance

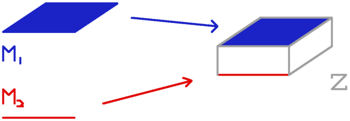
The *Gromov-Hausdorff distance* between compact spaces M_i is:

$$d_{GH}(M_1, M_2) = \inf \left\{ d_H^Z \left(\varphi_1(M_1), \varphi_2(M_2)\right) \mid \varphi_i : M_i \to Z \right\}$$

where the infimum is taken over all compact metric spaces, Z,
and over all distance preserving maps $\varphi_i : M_i^m \to Z$.

Here: $d_H^Z \left(\varphi_1(M_1), \varphi_2(M_2)\right)$ is the Hausdorff distance:

$$= \inf \left\{ R : \ \varphi_1(M_1) \subset T_R(\varphi_2(M_2)), \ \varphi_2(M_2) \subset T_R(\varphi_1(M_1)) \right\}$$



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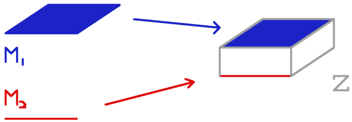

Gromov: $d_{GH}(M_2, M_1) < \epsilon \implies \exists\, 2\epsilon$-almost isom $\psi : M_2 \to M_1$.

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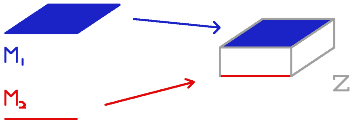

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Hint: Let $\psi(p)$ be a point $q \in M_1$ s.t. $d_Z(\varphi_2(p), \varphi_1(q))$ is min.

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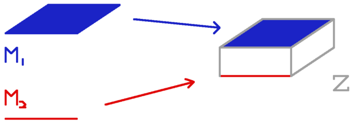
where the infimum is taken over all compact metric spaces, Z,

and over all distance preserving maps $\varphi_i : M_i^m \to Z$.

Here: $d_H^Z\left(\varphi_1(M_1), \varphi_2(M_2)\right)$ is the Hausdorff distance:

$$= \inf \{R : \varphi_1(M_1) \subset T_R(\varphi_2(M_2)),\ \varphi_2(M_2) \subset T_R(\varphi_1(M_1))\}$$



Gromov: $d_{GH}(M_2, M_1) < \epsilon \implies \exists\, 2\epsilon$-almost isom $\psi : M_2 \to M_1$.

Hint: Let $\psi(p)$ be a point $q \in M_1$ s.t. $d_Z(\varphi_2(p), \varphi_1(q))$ is min.

Gromov: $d_{GH}(M_2, M_1) = 0 \implies \exists$ an isometry $\psi : M_2 \to M_1$.

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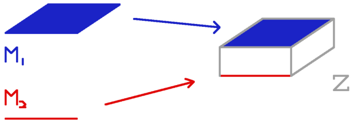

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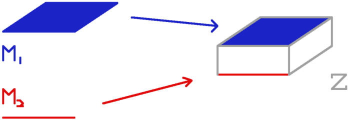

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Gromov: $\exists\, \epsilon$-alm isom $\psi_j : M_\infty \to M_j \implies d_{GH}(M_j, M_\infty) < 2\epsilon$.
Must construct a metric space Z. Can use nets with bridges.

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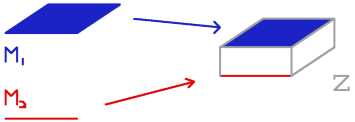

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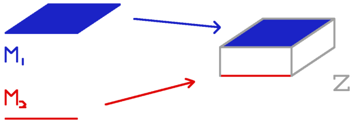

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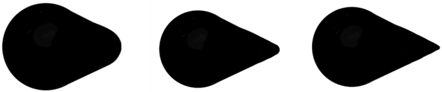
Thus, there is a uniform $N(r)$ = number of pts in an r-net of M_j.

More Sequences of Riemannian Manifolds



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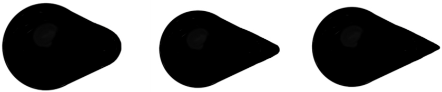

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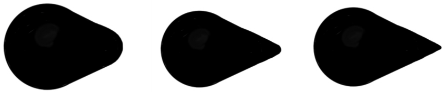

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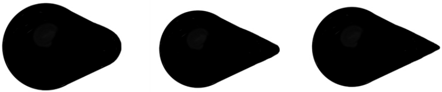

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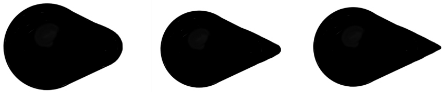

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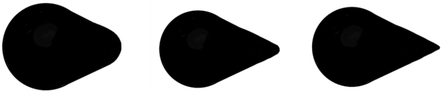

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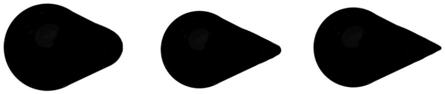

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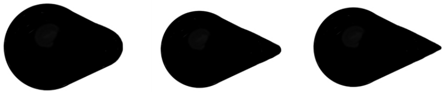

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Ilmanen: $\exists$ spheres M^3 with $Scalar > 0$ and inc many wells!!!
So GH only works well for $Ric \geq H$. We need a new convergence!!!

GH as Intrinsic Hausdorff Distance

Recall: The *Gromov-Hausdorff distance* between M_i^m is:

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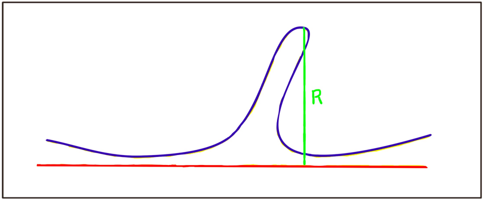
where **R** is as large as the depth of a well:



Sormani-Wenger: Intrinsic Flat Distance

The *intrinsic flat distance* between oriented manifolds M_i^m is:

$$d_{SWIF}(M_1^m, M_2^m) = \inf \left\{ d_F^Z\left(\varphi_{1\#}[[M_1^m]], \varphi_{2\#}[[M_2^m]]\right) \mid \varphi_i : M_i^m \to Z \right\}$$

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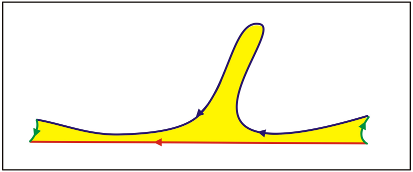
$$= \inf \left\{ \underset{length}{\mathbf{M}}(A) + \underset{area}{\mathbf{M}}(B) : A + \partial B = \varphi_{1\#}[[M_1^m]] - \varphi_{2\#}[[M_2^m]] \right\}$$



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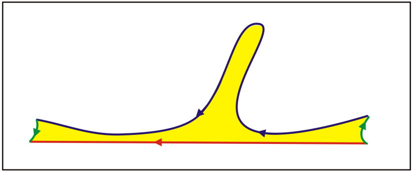

This is defined rigorously using [Ambrosio-Kirchheim] for
$M_j = (X_j, d_j, T_j)$ which are metric spaces with biLipschitz charts
and an integral current structure T_j such that $\mathrm{set}(T_j) = X_j$.
Ambriosio-Kirchheim Theory will be covered in Lecture 3.

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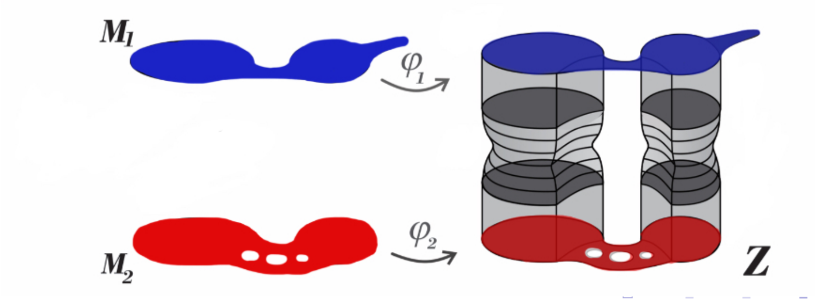

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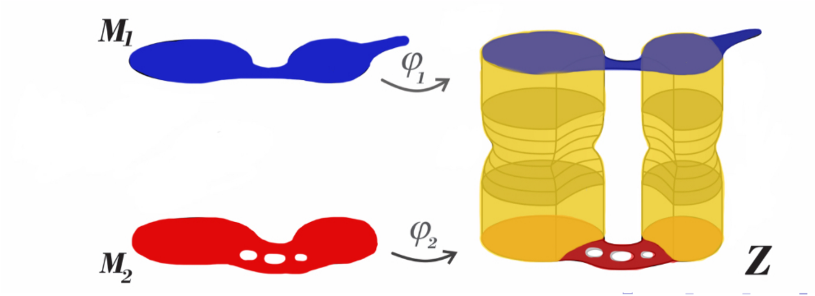

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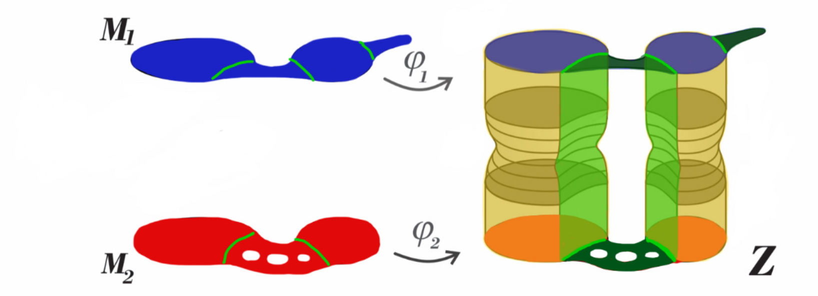
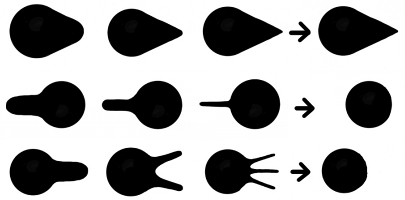


Sormani-Wenger: Intrinsic Flat Limits

Defn [SW]: $M_j \xrightarrow{\text{SWIF}} M_{SWIF}$ iff $d_{SWIF}(M_j, M_{SWIF}) \to 0$.



Observe how regions of small volume disappear.

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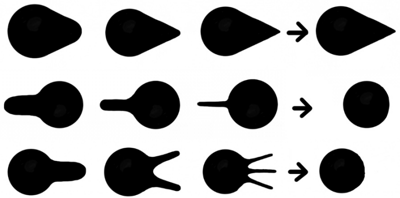

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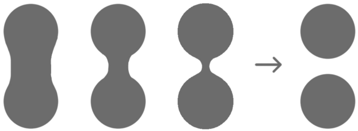
So the limits may not be connected metric spaces:



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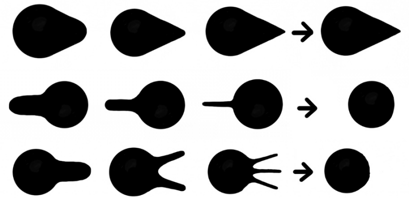

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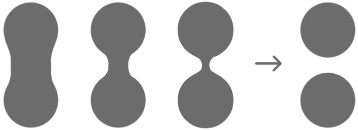
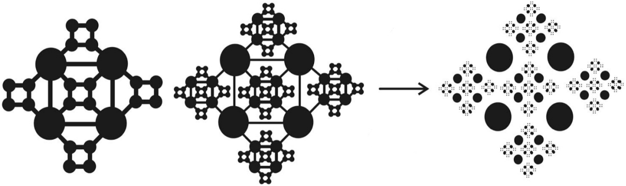


The limit spaces are called integral current spaces (X, d, T): they have countably many biLip charts and a notion of integration over those charts called the integral current structure, T.

Sormani-Wenger: Integral Current Spaces

A sequence of compact Riemannian manifolds can converge in the intrinsic flat (SWIF) sense to the following limit which is an integral current space:



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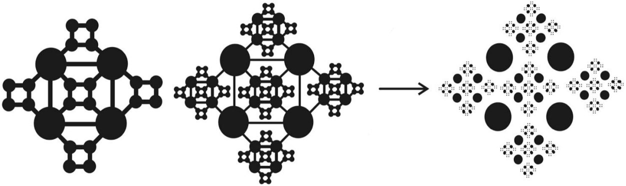

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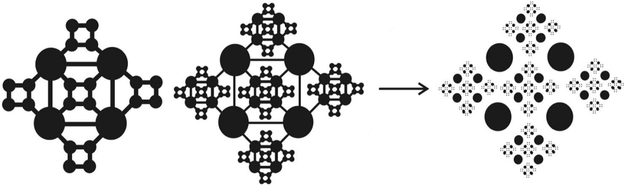

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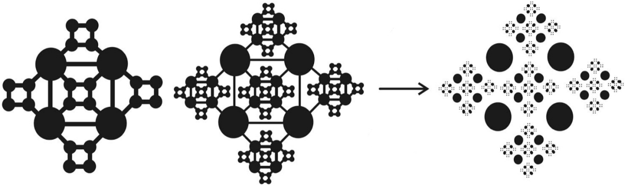

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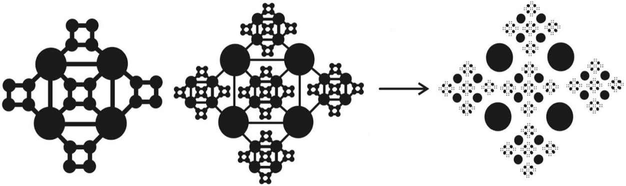

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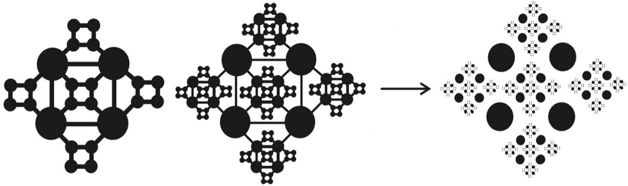

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Sormani-Wenger: Intrinsic Flat Distance

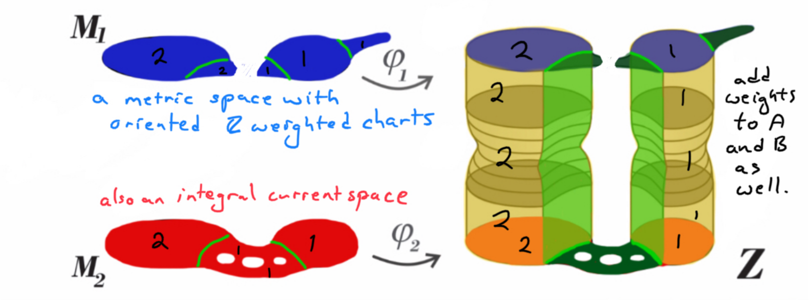
The *intrinsic flat distance* between integral current spaces $M_i^m = (X_i, d_i, T_i)$ which we will define carefully in Lecture 3 is:

$$d_{SWIF}(M_1^m, M_2^m) = \inf \left\{ d_F^Z \left(\varphi_{1\#} T_1, \varphi_{2\#} T_2 \right) \mid \varphi_i : M_i^m \to Z \right\}$$

where the infimum is taken over all complete metric spaces, Z, and over all distance preserving maps $\varphi_i : M_i^m \to Z$.

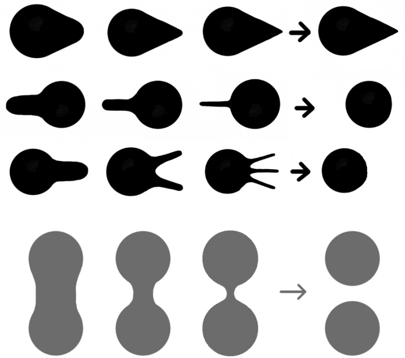
Here: $d_F^Z \left(\varphi_{1\#} T_1, \varphi_{2\#} T_2 \right)$ is the Federer-Fleming Flat dist

$$= \inf \left\{ \underset{area}{\mathbf{M}}(A) + \underset{vol}{\mathbf{M}}(B) : A + \partial B = \varphi_{1\#} T_1 - \varphi_{2\#} T_2 \right\}$$



More Intrinsic Flat Limits

Defn [SW]: $M_j \xrightarrow{\text{SWIF}} M_{SWIF}$ iff $d_{SWIF}(M_j, M_{SWIF}) \to 0$ where M_∞ is an integral current space.



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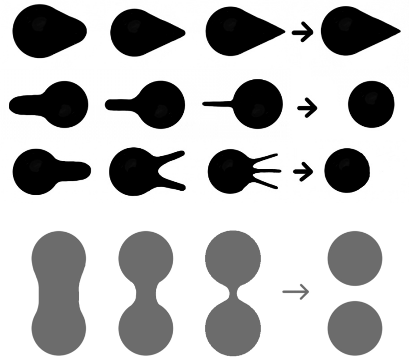

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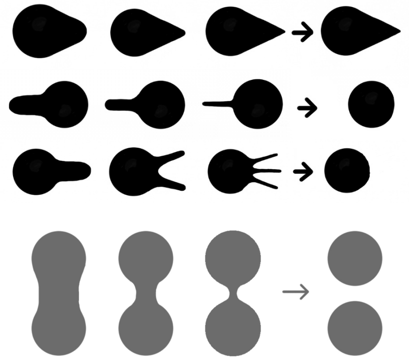

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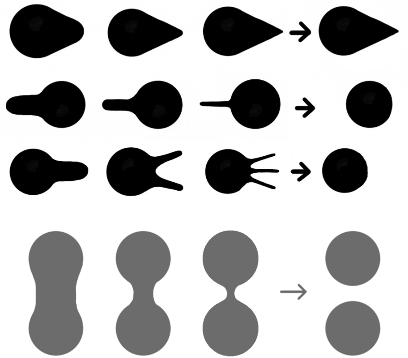

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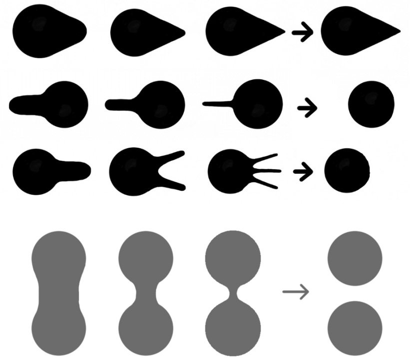

What about collapsing tori??
What about spheres shrinking to a point?
If $Vol(M_j) \to 0$ does the sequence disappear???
We say $M_j^m \xrightarrow{\text{SWIF}} 0^m$ where 0^m is the zero integral current space.

SWIF convergence to the 0^m space

Defn [SW]: We say $M_j \xrightarrow{\text{SWIF}} 0$ iff $d_{SWIF}(M_j^m, 0^m) \to 0$ where

$$d_{SWIF}(M_j^m, 0^m) = \inf \left\{ d_F^Z \left(\varphi_{1\#}[[M_1^m]], [[0]] \right) \mid \varphi_j : M_j^m \to Z \right\}$$

where the infimum is taken over all complete metric spaces, Z, and over all distance preserving maps $\varphi_j : M_j^m \to Z$.

Here: $d_F^Z \left(\varphi_{j\#}[[M_j^m]], [[0]] \right)$ is Federer-Fleming Flat dist:

$$= \inf \left\{ \mathbf{M}(A) + \mathbf{M}(B) : A + \partial B = \varphi_{j\#}[[M_j^m]] - [[0]] \right\}$$

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Pf: Take $Z = M_j$, $\varphi_j = id$, $A = \varphi_{j\#}[[M_j^m]]$, and $B = 0$. $\square$

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In Lecture 3: this notation will be explained and this definition will be made rigorous using Ambrosio-Kirchheim Theory.

SWIF Compactness Theorems

Thm [SW]: If $M_j \xrightarrow{\text{GH}} M_{GH}$ and $\text{Vol}(M_j) \le V_0$ and $\text{Vol}(\partial M_j) \le A_0$ then $\exists M_{j_k} \xrightarrow{\text{SWIF}} M_{SWIF}$ where $M_{SWIF} \subset M_{GH}$ or $M_{SWIF} = 0$.

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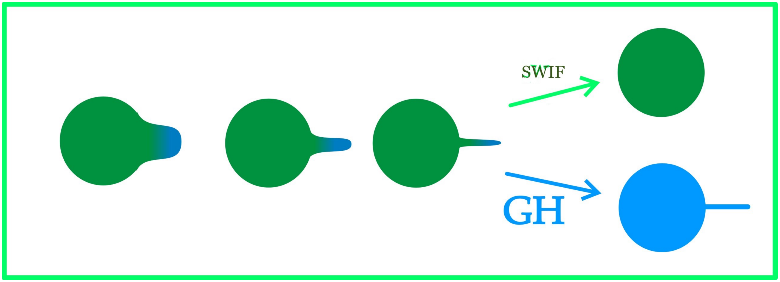

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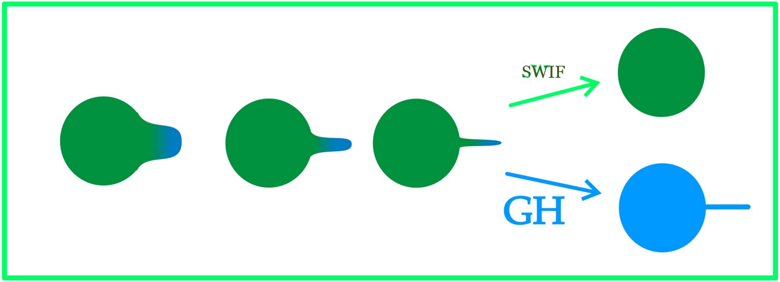

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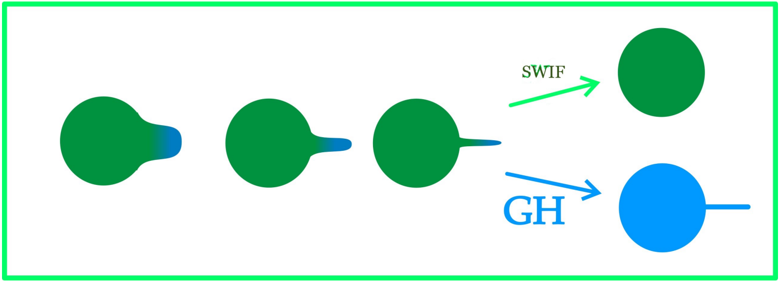

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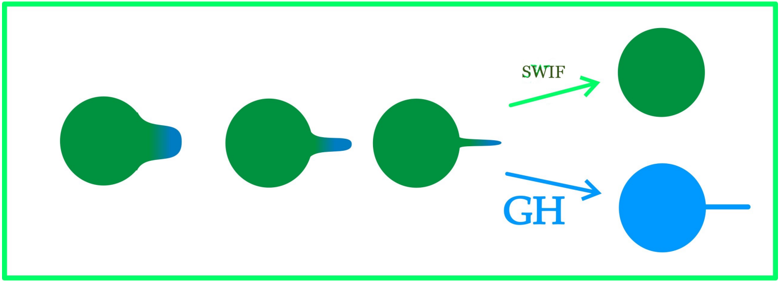

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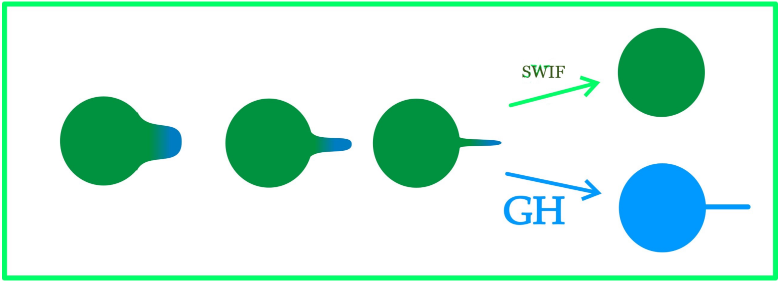

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In Lectures 3&4 we will rigorously define integral current spaces,
their masses, SWIF and $\mathcal{VF}$ convergence,
and techniques for proving the open problems.

Defns of $\mathcal{VF}$ and VADB Convergence

Defn: *Volume Preserving Intrinsic Flat Conv*: $M_j \xrightarrow{\mathcal{VF}} M_\infty$

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Allen-Perales-Sormani: [arXiv:2003.01172]

$$M_j \xrightarrow{\text{VADB}} M_\infty \implies M_j \xrightarrow{\mathcal{VF}} M_\infty.$$

Defn: *Volume Above Distance Below Conv:* $M_j \xrightarrow{\text{VADB}} M_\infty$

if $Vol_j(M_j) \to Vol_\infty(M_\infty)$ and $\exists D > 0$ s.t. $Diam(M_j) \le D$ and $\exists\ C^1\, \psi_j : M_\infty \to M_j$ s.t. $d_j(\psi_j(p), \psi_j(q)) \ge d_\infty(p, q)\ \forall p, q \in M_\infty$.

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Open Question: If $Scal_j \ge 0$ and $M_j \xrightarrow{\text{VADB}} M_\infty$ then
which properties of nonnegative scalar curvature hold on M_∞?

Review of Lecture I Notions of Convergence

$$M_j \xrightarrow{C^k} M_\infty \implies M_j \xrightarrow{C^0} M_\infty \implies M_j \xrightarrow{\mathrm{Lip}} M_\infty \implies M_j \xrightarrow{\mathrm{GH}} M_\infty$$

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Sormani-Wenger Compactness:

$$M_j \xrightarrow{\text{GH}} M_{GH} \text{ and } Vol(M_j) \leq V \implies \exists M_{j_k} \xrightarrow{\text{SWIF}} M_{SWIF} \subset M_{GH}$$

Review of Lecture I Notions of Convergence

$$M_j \xrightarrow{C^k} M_\infty \implies M_j \xrightarrow{C^0} M_\infty \implies M_j \xrightarrow{\text{Lip}} M_\infty \implies M_j \xrightarrow{\text{GH}} M_\infty$$

Gromov Compactness:

$$Diam(M_j) \le D \text{ and } \text{Ricci}_j \ge H \implies \exists M_{j_k} \xrightarrow{\text{GH}} M_{GH}$$

Sormani-Wenger Compactness:

$$M_j \xrightarrow{\text{GH}} M_{GH} \text{ and } Vol(M_j) \le V \implies \exists M_{j_k} \xrightarrow{\text{SWIF}} M_{SWIF} \subset M_{GH}$$

Sormani-Wenger, Matveev-Portegies:

$$Vol(M_j) \ge V_1 \text{ and } \text{Ricci}_j \ge H \implies M_{SWIF} = M_{GH}$$

Review of Lecture I Notions of Convergence

$$M_j \xrightarrow{C^k} M_\infty \implies M_j \xrightarrow{C^0} M_\infty \implies M_j \xrightarrow{\text{Lip}} M_\infty \implies M_j \xrightarrow{\text{GH}} M_\infty$$

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$$Diam(M_j) \le D \text{ and } \text{Ricci}_j \ge H \implies \exists M_{j_k} \xrightarrow{\text{GH}} M_{GH}$$

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$$M_j \xrightarrow{\text{GH}} M_{GH} \text{ and } Vol(M_j) \le V \implies \exists M_{j_k} \xrightarrow{\text{SWIF}} M_{SWIF} \subset M_{GH}$$

Sormani-Wenger, Matveev-Portegies:

$$Vol(M_j) \ge V_1 \text{ and } \text{Ricci}_j \ge H \implies M_{SWIF} = M_{GH}$$

Wenger Compactness:

$$Diam(M_j) \le D \text{ and } Vol(M_j) \le V \implies \exists M_{j_k} \xrightarrow{\text{SWIF}} M_{SWIF}$$

Review of Lecture I Notions of Convergence

$$M_j \xrightarrow{C^k} M_\infty \implies M_j \xrightarrow{C^0} M_\infty \implies M_j \xrightarrow{\text{Lip}} M_\infty \implies M_j \xrightarrow{\text{GH}} M_\infty$$

Gromov Compactness:

$$Diam(M_j) \le D \text{ and } \text{Ricci}_j \ge H \implies \exists M_{j_k} \xrightarrow{\text{GH}} M_{GH}$$

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$$M_j \xrightarrow{\text{GH}} M_{GH} \text{ and } Vol(M_j) \le V \implies \exists M_{j_k} \xrightarrow{\text{SWIF}} M_{SWIF} \subset M_{GH}$$

Sormani-Wenger, Matveev-Portegies:

$$Vol(M_j) \ge V_1 \text{ and } \text{Ricci}_j \ge H \implies M_{SWIF} = M_{GH}$$

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$$Diam(M_j) \le D \text{ and } Vol(M_j) \le V \implies \exists M_{j_k} \xrightarrow{\text{SWIF}} M_{SWIF}$$

Allen-Perales-Sormani:

$$M_j \xrightarrow{\text{Lip}} M \implies M_j \xrightarrow{\text{VADB}} M \implies M_j \xrightarrow{\mathcal{VF}} M \implies M_j \xrightarrow{\text{SWIF}} M$$

Review of Lecture I Notions of Convergence

$$M_j \xrightarrow{C^k} M_\infty \implies M_j \xrightarrow{C^0} M_\infty \implies M_j \xrightarrow{\text{Lip}} M_\infty \implies M_j \xrightarrow{\text{GH}} M_\infty$$

Gromov Compactness:

$$Diam(M_j) \le D \text{ and } \text{Ricci}_j \ge H \implies \exists M_{j_k} \xrightarrow{\text{GH}} M_{GH}$$

Sormani-Wenger Compactness:

$$M_j \xrightarrow{\text{GH}} M_{GH} \text{ and } Vol(M_j) \le V \implies \exists M_{j_k} \xrightarrow{\text{SWIF}} M_{SWIF} \subset M_{GH}$$

Sormani-Wenger, Matveev-Portegies:

$$Vol(M_j) \ge V_1 \text{ and } \text{Ricci}_j \ge H \implies M_{SWIF} = M_{GH}$$

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$$M_j \xrightarrow{\text{Lip}} M \implies M_j \xrightarrow{\text{VADB}} M \implies M_j \xrightarrow{\mathcal{VF}} M \implies M_j \xrightarrow{\text{SWIF}} M$$

Gromov-Sormani IAS Scalar Compactness Conjecture:

$$Diam_j \le D,\ Vol_j \le V,\ Scal_j \ge 0,\ MinA_j \ge A \implies \exists M_{j_k} \xrightarrow{\mathcal{VF}} M_{SWIF}$$

Intrinsic Flat and Gromov-Hausdorff Convergence

Lecture 1: Geometric Notions of Convergence DONE!

$$M_j \xrightarrow{C^k} M_\infty \implies M_j \xrightarrow{C^0} M_\infty \implies M_j \xrightarrow{\text{Lip}} M_\infty \implies M_j \xrightarrow{\text{GH}} M_\infty$$

$$M_j \xrightarrow{\text{Lip}} M \implies M_j \xrightarrow{\text{VADB}} M \implies M_j \xrightarrow{\mathcal{VF}} M \implies M_j \xrightarrow{\text{SWIF}} M$$

Intrinsic Flat and Gromov-Hausdorff Convergence

Lecture 1: Geometric Notions of Convergence DONE!

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[Gromov Structures-Metriques] [Burago-Burago-Ivanov Text]
[Sormani-Wenger JDG-2011] [Allen-Perales-Sormani VADB]
[Conjectures on Convergence and Scalar arXiv: 2103.10093]

Intrinsic Flat and Gromov-Hausdorff Convergence

Lecture 1: Geometric Notions of Convergence DONE!

$$M_j \xrightarrow{C^k} M_\infty \implies M_j \xrightarrow{C^0} M_\infty \implies M_j \xrightarrow{\text{Lip}} M_\infty \implies M_j \xrightarrow{\text{GH}} M_\infty$$

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[Gromov Structures-Metriques] [Burago-Burago-Ivanov Text]
[Sormani-Wenger JDG-2011] [Allen-Perales-Sormani VADB]
[Conjectures on Convergence and Scalar arXiv: 2103.10093]

Lecture 2: Open Problems about Scalar Curvature NEXT!

Intrinsic Flat and Gromov-Hausdorff Convergence

Lecture 1: Geometric Notions of Convergence DONE!

$$M_j \xrightarrow{C^k} M_\infty \implies M_j \xrightarrow{C^0} M_\infty \implies M_j \xrightarrow{\text{Lip}} M_\infty \implies M_j \xrightarrow{\text{GH}} M_\infty$$

$$M_j \xrightarrow{\text{Lip}} M \implies M_j \xrightarrow{\text{VADB}} M \implies M_j \xrightarrow{\mathcal{VF}} M \implies M_j \xrightarrow{\text{SWIF}} M$$

[Gromov Structures-Metriques] [Burago-Burago-Ivanov Text]
[Sormani-Wenger JDG-2011] [Allen-Perales-Sormani VADB]
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Lecture 2: Open Problems about Scalar Curvature NEXT!

$\mathcal{VF}$ Scalar Compactness Conjecture

Intrinsic Flat and Gromov-Hausdorff Convergence

Lecture 1: Geometric Notions of Convergence DONE!

$$M_j \xrightarrow{C^k} M_\infty \implies M_j \xrightarrow{C^0} M_\infty \implies M_j \xrightarrow{\text{Lip}} M_\infty \implies M_j \xrightarrow{\text{GH}} M_\infty$$

$$M_j \xrightarrow{\text{Lip}} M \implies M_j \xrightarrow{\text{VADB}} M \implies M_j \xrightarrow{\mathcal{VF}} M \implies M_j \xrightarrow{\text{SWIF}} M$$

[Gromov Structures-Metriques] [Burago-Burago-Ivanov Text]
[Sormani-Wenger JDG-2011] [Allen-Perales-Sormani VADB]
[Conjectures on Convergence and Scalar arXiv: 2103.10093]

Lecture 2: Open Problems about Scalar Curvature NEXT!

$\mathcal{VF}$ Scalar Compactness Conjecture

$\mathcal{VF}$ Almost Rigidity of the Positive Mass Theorem

Intrinsic Flat and Gromov-Hausdorff Convergence

Lecture 1: Geometric Notions of Convergence DONE!

$$M_j \xrightarrow{C^k} M_\infty \implies M_j \xrightarrow{C^0} M_\infty \implies M_j \xrightarrow{\text{Lip}} M_\infty \implies M_j \xrightarrow{\text{GH}} M_\infty$$

$$M_j \xrightarrow{\text{Lip}} M \implies M_j \xrightarrow{\text{VADB}} M \implies M_j \xrightarrow{\mathcal{VF}} M \implies M_j \xrightarrow{\text{SWIF}} M$$

[Gromov Structures-Metriques] [Burago-Burago-Ivanov Text]
[Sormani-Wenger JDG-2011] [Allen-Perales-Sormani VADB]
[Conjectures on Convergence and Scalar arXiv: 2103.10093]

Lecture 2: Open Problems about Scalar Curvature NEXT!

$\mathcal{VF}$ Scalar Compactness Conjecture
$\mathcal{VF}$ Almost Rigidity of the Positive Mass Theorem
$\mathcal{VF}$ Geometric Stability of the Scalar Torus Rigidity and

Intrinsic Flat and Gromov-Hausdorff Convergence

Lecture 1: Geometric Notions of Convergence DONE!

$$M_j \xrightarrow{C^k} M_\infty \implies M_j \xrightarrow{C^0} M_\infty \implies M_j \xrightarrow{\text{Lip}} M_\infty \implies M_j \xrightarrow{\text{GH}} M_\infty$$

$$M_j \xrightarrow{\text{Lip}} M \implies M_j \xrightarrow{\text{VADB}} M \implies M_j \xrightarrow{\mathcal{VF}} M \implies M_j \xrightarrow{\text{SWIF}} M$$

[Gromov Structures-Metriques] [Burago-Burago-Ivanov Text]
[Sormani-Wenger JDG-2011] [Allen-Perales-Sormani VADB]
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Lecture 2: Open Problems about Scalar Curvature NEXT!

$\mathcal{VF}$ Scalar Compactness Conjecture
$\mathcal{VF}$ Almost Rigidity of the Positive Mass Theorem
$\mathcal{VF}$ Geometric Stability of the Scalar Torus Rigidity and
Scalar Sphere Rigidity and Prism Rigidity Theorems

Intrinsic Flat and Gromov-Hausdorff Convergence

Lecture 1: Geometric Notions of Convergence DONE!

$$M_j \xrightarrow{C^k} M_\infty \implies M_j \xrightarrow{C^0} M_\infty \implies M_j \xrightarrow{\text{Lip}} M_\infty \implies M_j \xrightarrow{\text{GH}} M_\infty$$

$$M_j \xrightarrow{\text{Lip}} M \implies M_j \xrightarrow{\text{VADB}} M \implies M_j \xrightarrow{\mathcal{VF}} M \implies M_j \xrightarrow{\text{SWIF}} M$$

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[Sormani-Wenger JDG-2011] [Allen-Perales-Sormani VADB]
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Lecture 2: Open Problems about Scalar Curvature NEXT!

$\mathcal{VF}$ Scalar Compactness Conjecture
$\mathcal{VF}$ Almost Rigidity of the Positive Mass Theorem
$\mathcal{VF}$ Geometric Stability of the Scalar Torus Rigidity and
Scalar Sphere Rigidity and Prism Rigidity Theorems
Special Cases involving VADB and Volume Convergence

Intrinsic Flat and Gromov-Hausdorff Convergence

Lecture 1: Geometric Notions of Convergence DONE!

$$M_j \xrightarrow{C^k} M_\infty \implies M_j \xrightarrow{C^0} M_\infty \implies M_j \xrightarrow{\text{Lip}} M_\infty \implies M_j \xrightarrow{\text{GH}} M_\infty$$

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Lecture 2: Open Problems about Scalar Curvature NEXT!

$\mathcal{VF}$ Scalar Compactness Conjecture
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Scalar Sphere Rigidity and Prism Rigidity Theorems
Special Cases involving VADB and Volume Convergence
will be assigned to teams of interested students/postdocs

[Conjectures on Convergence and Scalar arXiv: 2103.10093]

Other papers on SWIF convergence may be found at:
https://sites.google.com/site/intrinsicflatconvergence/

This talk may be downloaded at my website:
https://sites.google.com/site/professorsormani/

.

Feel free to email me with questions: sormanic@gmail.com

Thank you for listening - Christina Sormani