Intrinsic Flat and Gromov-Hausdorff Convergence

Christina Sormani

CUNY GC and Lehman College

Lecture II: Open Problems on Scalar Curvature

Intrinsic Flat and Gromov-Hausdorff Convergence

Lecture 1: Geometric Notions of Convergence DONE!

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Lectures 3&4: Techniques to Apply to Prove Convergence

See https://sites.google.com/site/intrinsicflatconvergence/

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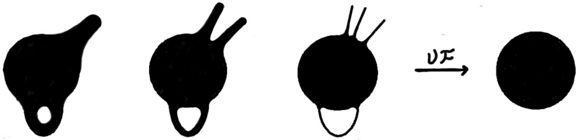

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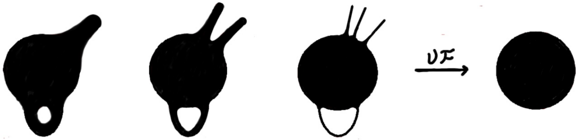

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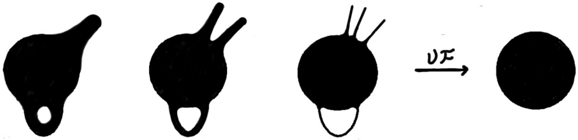

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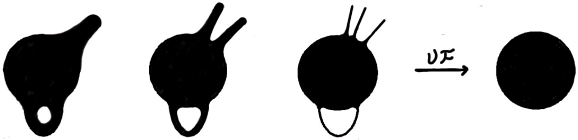

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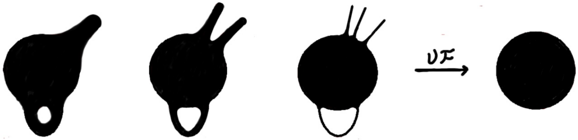

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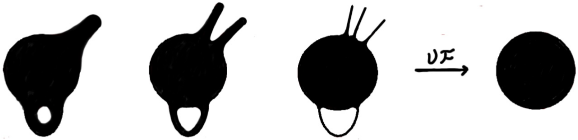

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Let's quickly review these notions of convergence...

Volume Preserving Intrinsic Flat $\mathcal{VF}$ Convergence

Defn: $M_j \xrightarrow{\mathcal{VF}} M_\infty$ if $M_j \xrightarrow{\mathcal{F}} M_\infty$ and $\mathrm{Vol}(M_j) \to \mathrm{Vol}(M_\infty)$.

Defn: $M_j \xrightarrow{\mathcal{F}} M_\infty$ if $d_{\mathcal{F}}(M_j, M_\infty) = d_{SWIF}(M_j, M_\infty) \to 0$:

Sormani-Wenger: Intrinsic Flat Distance

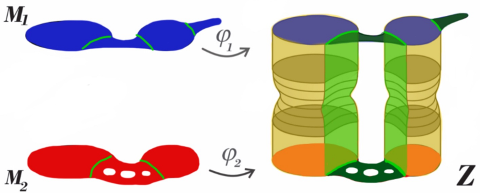
The *intrinsic flat distance* between oriented manifolds M_i^m is:

$$d_{SWIF}(M_1^m, M_2^m) = \inf\left\{ d_F^Z\left(\varphi_{1\#}[[M_1^m]], \varphi_{2\#}[[M_2^m]]\right) \mid \varphi_i : M_i^m \to Z \right\}$$

where the infimum is taken over all complete metric spaces, Z, and over all distance preserving maps $\varphi_i : M_i^m \to Z$.

Here: $d_F^Z\left(\varphi_{1\#}[[M_1^m]], \varphi_{2\#}[[M_2^m]]\right)$ is the Federer-Fleming Flat dist

$$= \inf\left\{ \underset{area}{\mathbf{M}}(A) + \underset{vol}{\mathbf{M}}(B) : A + \partial B = \varphi_{1\#}[[M_1^m]] - \varphi_{2\#}[[M_2^m]] \right\}$$



$$M_j \xrightarrow{C^k} M \implies M_j \xrightarrow{C^0} M \implies M_j \xrightarrow{\mathrm{VADB}} M \implies M_j \xrightarrow{\mathcal{VF}} M$$

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Defn: $M_j \xrightarrow{C^k} M_\infty$ if $\exists$ C^{k+1} $\psi_j : M_\infty \to M_j$ s.t. $\psi_j^* g_j \xrightarrow{C^k} g_\infty$:

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Defn: *Volume Above Distance Below Conv:* $M_j \xrightarrow{\mathrm{VADB}} M_\infty$
if $Vol_j(M_j) \to Vol_\infty(M_\infty)$ and $\exists D > 0$ s.t. $Diam(M_j) \le D$ and
$\exists$ C^1 $\psi_j : M_\infty \to M_j$ s.t. $d_j(\psi_j(p), \psi_j(q)) \ge d_\infty(p, q)$ $\forall p, q \in M_\infty$.

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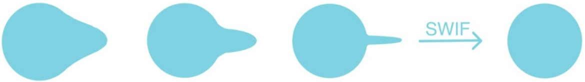

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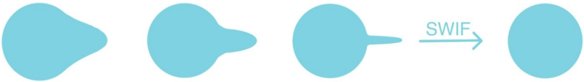

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Note: $\mathcal{VF}$ and *SWIF* do not require a $\psi_j : M_\infty \to M_j$!

Volume Limit Definition of Scalar Curvature

A compact manifold (M, g) with a C^0 metric tensor g has $d(p,q) = \inf\{L_g(C)\}$ where $L(C) = \int_0^1 g(C'(t), C'(t))^{1/2}\, dt.$

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Areas and Volumes can be defined using Hausdorff measure:

$$\mathrm{Vol}(U) = \mathcal{H}^n(U) = \lim_{\delta\to 0} \inf \left\{ c_n \sum_{j=1}^{\infty} (\mathrm{Diam}(U_j))^n \ :\ U \subset \bigcup_{j=1}^{\infty} U_j,\ \mathrm{Diam}(U_j) \le \delta \right\}.$$

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Scalar Curvature is defined

$$\mathrm{Scal}(p) = \lim_{r\to 0} 6(n+2)\left(\frac{V^n(r) - Vol(B(p,r))}{r^2 V^n(r)}\right)$$

where $V^n(r)$ is the volume of a Euclidean ball of radius r.

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where $V^n(r)$ is the volume of a Euclidean ball of radius r.
A *flat three torus* (whose small balls are Euclidean) has $Scal = 0$ and the *standard three sphere* $(\mathbb{S}^3, g_{\mathbb{S}^3})$ has $Scal = 6$ everywhere.

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A *flat three torus* (whose small balls are Euclidean) has $Scal = 0$ and the *standard three sphere* $(\mathbb{S}^3, g_{\mathbb{S}^3})$ has $Scal = 6$ everywhere.
Gromov: If M_j^3 have $\mathrm{Scal} \ge 0$, does M_∞ have $\mathrm{Scal} \ge 0$?

Schwarzschild Space and Ilmanen Wells

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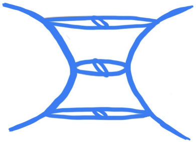
A *flat three torus* (whose small balls are Euclidean) has $Scal = 0$ and the *standard three sphere* $(\mathbb{S}^3, g_{\mathbb{S}^3})$ has $Scal = 6$ everywhere.
Riemannian Schwarzschild Space has $Scal = 0$.



Schwarzschild Space and Ilmanen Wells

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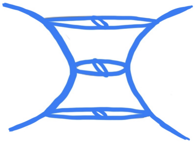
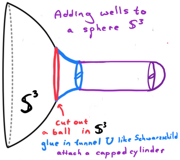
$$\mathrm{Scal}(p) = \lim_{r \to 0} 6(n+2) \left(\frac{V^n(r) - Vol(B(p,r))}{r^2 V^n(r)} \right)$$

where $V^n(r)$ is the volume of a Euclidean ball of radius r.
A *flat three torus* (whose small balls are Euclidean) has $Scal = 0$ and the *standard three sphere* $(\mathbb{S}^3, g_{\mathbb{S}^3})$ has $Scal = 6$ everywhere.
Riemannian Schwarzschild Space has $Scal = 0$.





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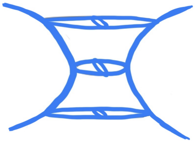

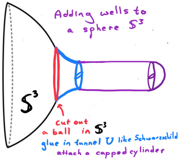

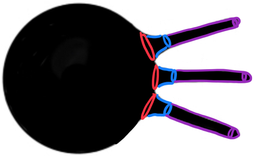

If we cut a ball out of a sphere and attach a Schwarzschild neck, and then a capped cylinder, and smooth it we get M^3 with a well that has Scal > 0. Adding many wells gives Ilmanen's Example.

Gromov-Lawson/Schoen-Yau Tunnels

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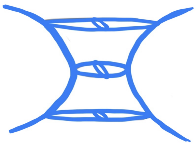

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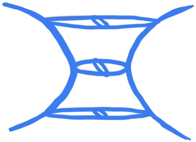

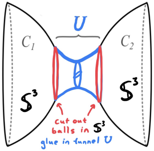

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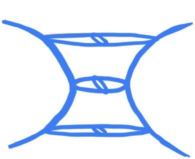

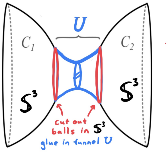

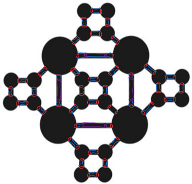

Gromov-Lawson/Schoen-Yau tunnels can be glued between spheres $\mathbb{S}^3$ to create connected sums of spheres with Scal > 0.

Gromov: If M_j have Scal ≥ 0 does M_∞ have Scal ≥ 0?

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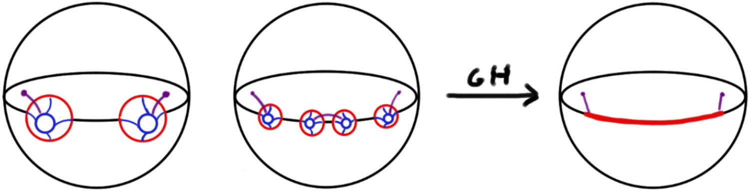
Basilio-Dodziuk-Sormani: Sewing tunnels along a curve $\eta \subset \mathbb{S}^3$:



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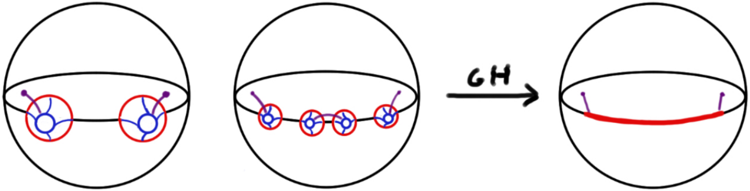

We construct M_j with $\mathrm{Scal}_j \geq 0$ and $M_j \xrightarrow{\mathrm{GH}} M_{GH}$

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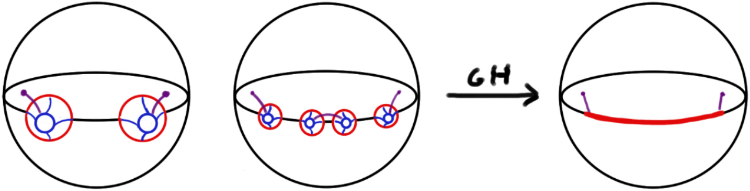

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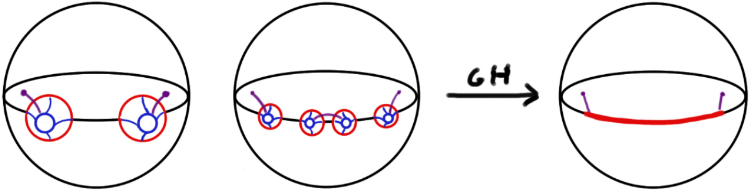

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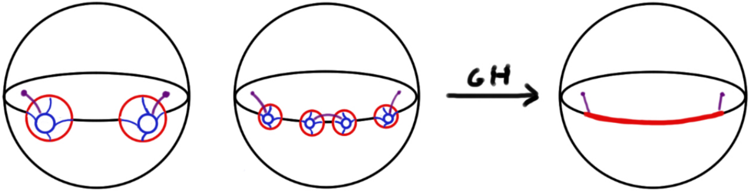

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Maybe other properties of Scalar curvature persist under limits?

$\mathrm{Scal}_j \geq 0$, $\mathrm{Vol}(M_j^3) \to \mathrm{Vol}(\mathbb{S}^3)$ but $M_j^3 \xrightarrow{\mathcal{VF}} 0^3$

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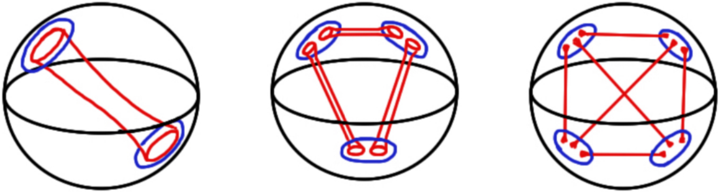
We take an increasingly dense collection of balls in $\mathbb{S}^3$



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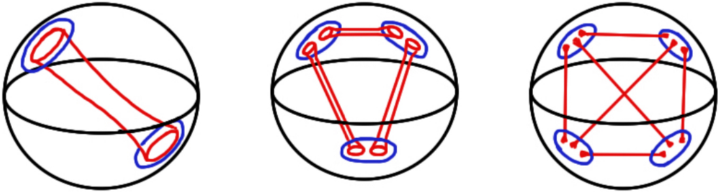

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run a tiny short tunnel

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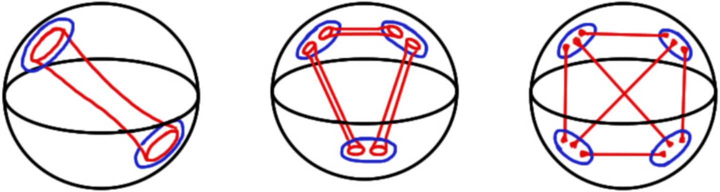

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Ensure that the total volume of the tunnels $\to 0$.

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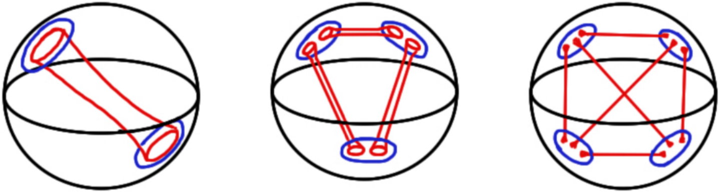

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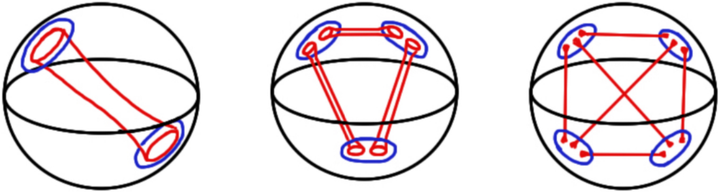

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We could also limit ourselves to balls in a domain U

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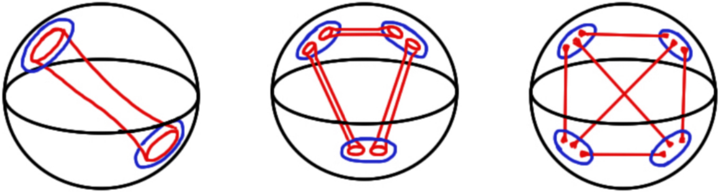

Between every pair of balls $B(p, r)$ and $B(q, r)$
run a tiny short tunnel
Ensure that the total volume of the tunnels $\to 0$.
$M_{GH} = \{p\}$ and so $M_{SWIF} = 0^3$.
We could also limit ourselves to balls in a domain U
then $M_{GH} = M_{SWIF} = \mathbb{S}^3|_U$ where U is identified to a point.

$\mathrm{Scal}_j \geq 0$, $M_j^3 \xrightarrow{\mathcal{VF}} M_\infty^3$, but M_∞^3 has no geods!

Basilio-Kazaras-Sormani:

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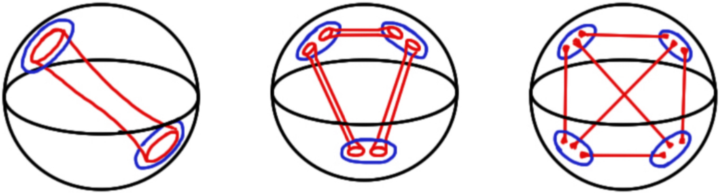

Between every pair of balls $B(p, r)$ and $B(q, r)$
run a tunnel of length$=d_{\mathbb{E}^4}(p, q)$

$\mathrm{Scal}_j \geq 0$, $M^3_j \xrightarrow{\mathcal{VF}} M^3_\infty$, but M^3_∞ has no geods!

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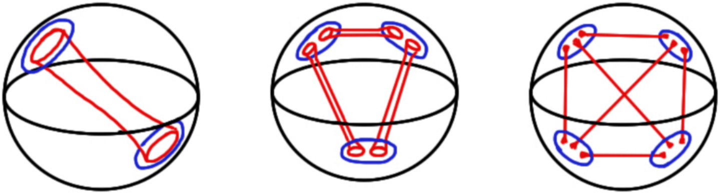

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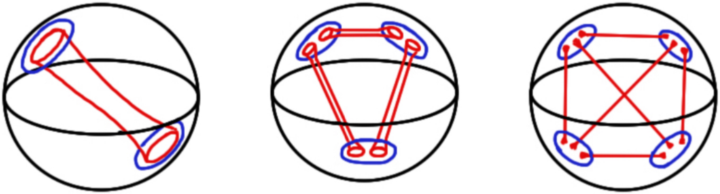

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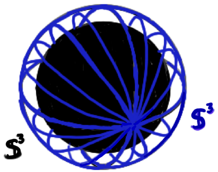
Ensure that the total volume of the tunnels $\to 0$.

$$M_j^3 \xrightarrow{\mathcal{VF}} M_\infty^3 = (\mathbb{S}^3, d_{\mathbb{E}^4}, [[\mathbb{S}^3]])$$

which has no geodesics.

Sormani-Wenger: Cancellation and Doubling ($\text{Scal}_j \geq 0$)



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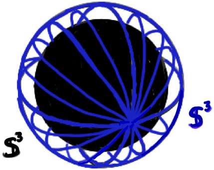

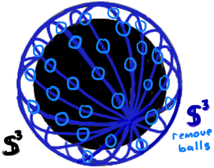

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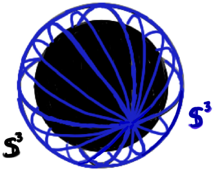

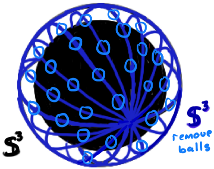

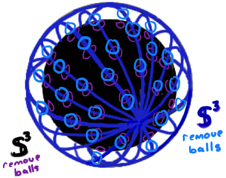

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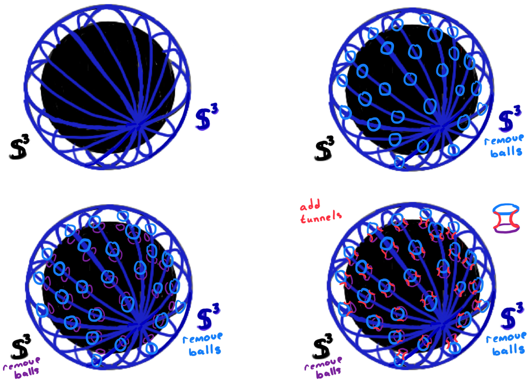

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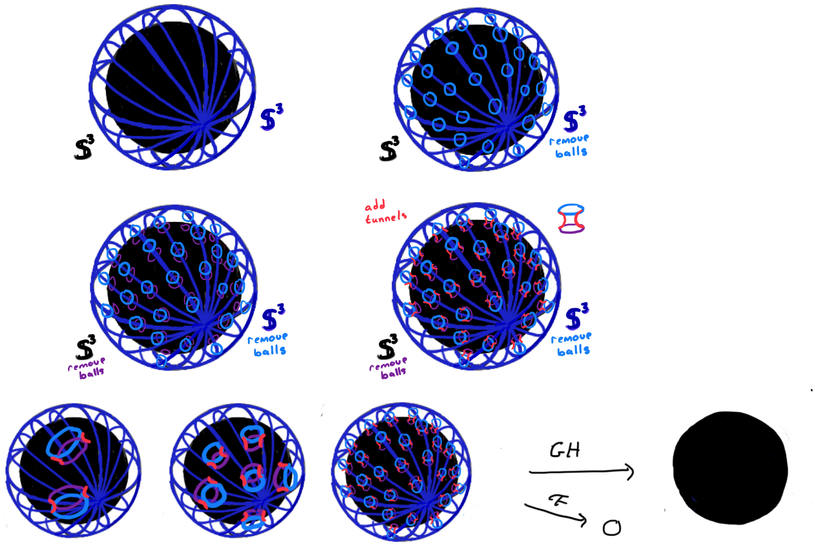

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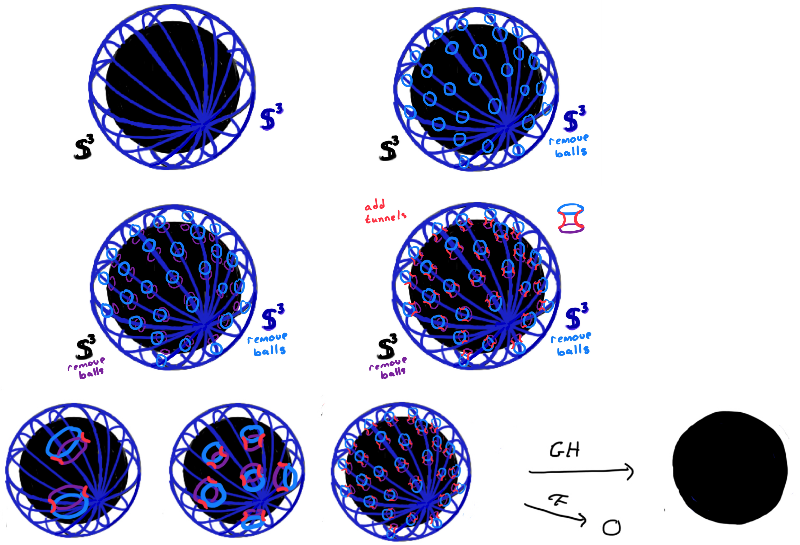
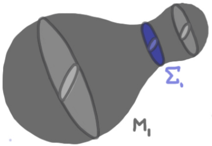


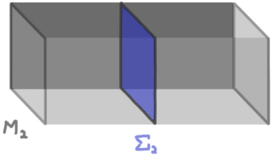
and if we twist each tunnel $M_j \xrightarrow{\text{SWIF}}$ ($\mathbb{S}^3$ with weight 2).

Stable Minimal Surfaces in M^3 with Scal ≥ 0

A surface $\Sigma^2 \subset M^3$ is a stable closed minimal surface if it minimizes area of any continuous deformation, Σ_t:

$$Area(\Sigma^2) \leq \text{Area}(\Sigma_t) \quad \forall \Sigma_t \ s.t. \ \Sigma_0 = \Sigma \text{ and } \partial\Sigma_t = \partial\Sigma_0.$$





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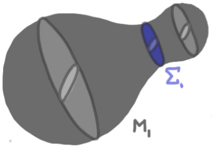

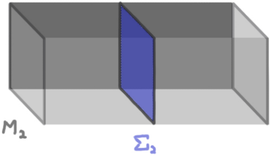

Schoen-Yau: If g is C^2 then by taking the second variation of the area and applying the Gauss-Bonnet Theorem, the only stable closed minimal surfaces in M^3 with Scal ≥ 0 are tori and spheres.

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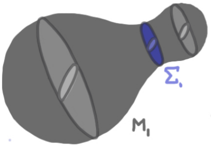

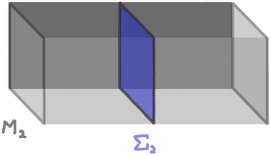

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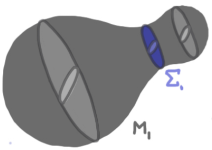

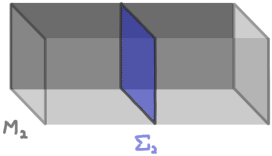

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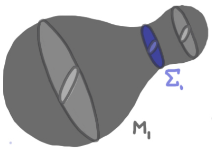

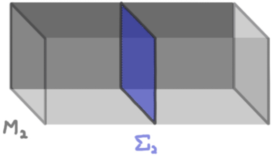

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Basilio-Sormani (to appear): sewing to M_{GH} is $\mathbb{T}^3$ with η to a p.

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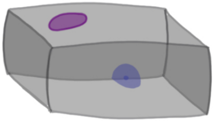

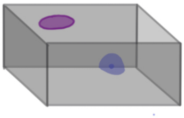

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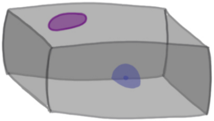

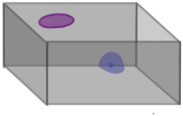

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where the infimum is over all sweepouts Σ_t of M^3:

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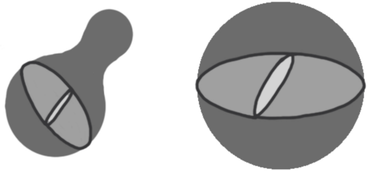

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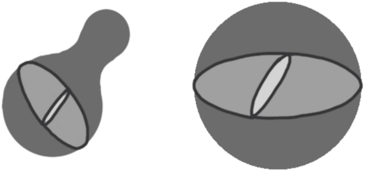

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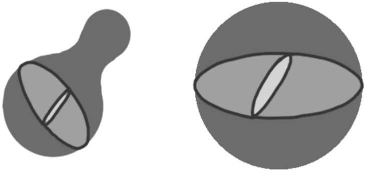

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Do Scalar Rigidity Theorems hold on Limit Spaces?

Basilio-Dodziuk-Sormani: some sewing nonexamples for GH limits.

Now lets consider:

$$M_j \xrightarrow{C^k} M \implies M_j \xrightarrow{C^0} M \implies M_j \xrightarrow{\text{VADB}} M \implies M_j \xrightarrow{\mathcal{VF}} M$$

Defn: $M_j \xrightarrow{C^k} M_\infty$ if $\exists$ C^{k+1} $\psi_j : M_\infty \to M_j$ s.t. $\psi_j^* g_j \xrightarrow{C^k} g_\infty$:

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Prism Rigidity Theorem of Gromov-Li
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See [Conjectures on Conv and Scalar (2021) arXiv: 2103.10093]

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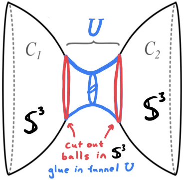

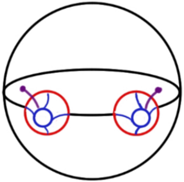

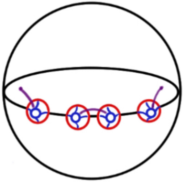

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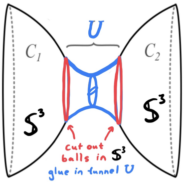

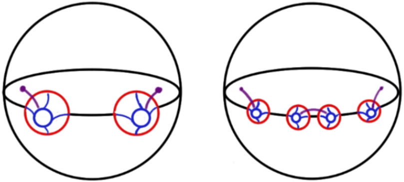

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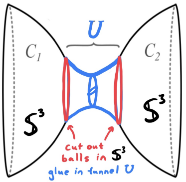

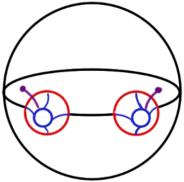

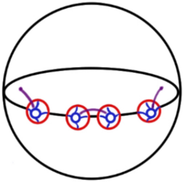

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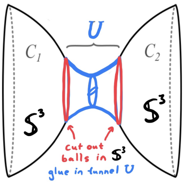

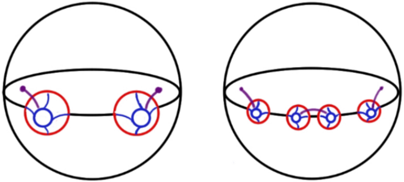

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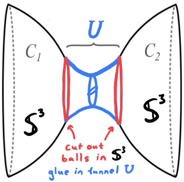

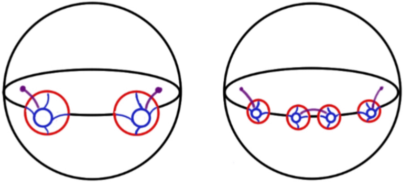

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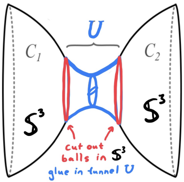

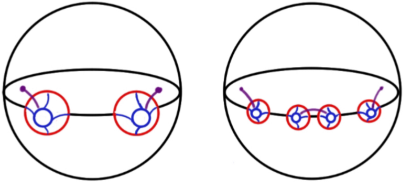

Their topology is increasing so M_j do not conv in VADB sense.

Notice each tunnel contains a stable minimal surface, Σ^2.
As we add more tiny tunnels, $MinA(M_j) \to 0$, where

$$MinA(M) = \min\{Area(\Sigma^2) : \Sigma^2 \subset M^3 \text{ a min surf}\} .$$

Do Scalar Rigidity Theorems hold on Limit Spaces M_∞
if we assume $M_j \xrightarrow{\mathcal{VF}} M_\infty$ with $MinA(M_j) \geq A > 0$?

These hypotheses eliminate all known nonexamples! OPEN

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A second method to prove these:
(1) prove the IAS Scalar Compactness Conj [Gromov-Sormani]
to show that at least a subsequence $M_{j_k} \xrightarrow{\mathcal{VF}} M_\infty$.
(2) prove each rigidity theorem holds on limits M_∞ where

$$M_j \xrightarrow{\mathcal{VF}} M_\infty \text{ with } MinA(M_j) \geq A > 0$$

Due to the Nonexamples of Basilio-Dodziuk-Sormani
we require the $MinA(M)$ hypotheses in these four conjectures.

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Review: What properties of scalar curvature are conserved when $M_j \xrightarrow{C^0} M_\infty$? Many already proven by Gromov, Bamler, Li, Burkhardt-Guim, and others.

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Other warped products being studied by another team.

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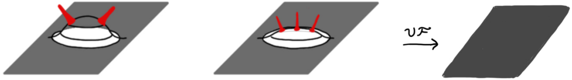
Conjecture: Suppose M_j^3 are homeomorphic to tori with

$$\mathrm{Scal}_j \geq -1/j \quad MinA_j \geq \alpha > 0 \quad \mathrm{Diam}_j \leq D \quad \mathrm{Vol}_j \leq V$$

then a subseq $M_{j_k}^3 \xrightarrow{\mathcal{VF}} M_\infty$ where M_∞ is a flat torus.



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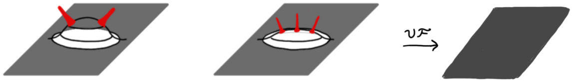

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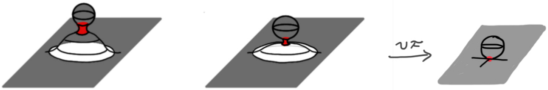

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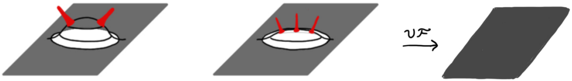

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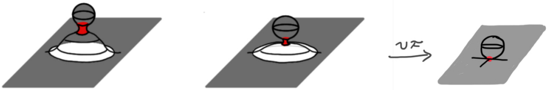

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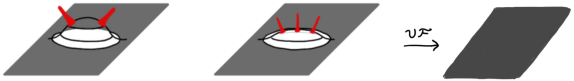

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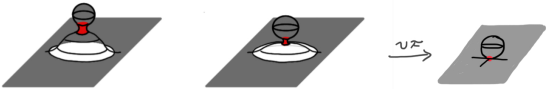

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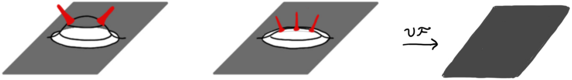

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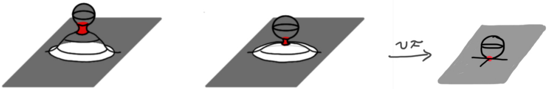

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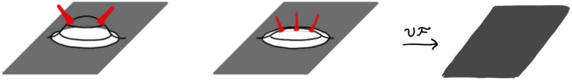

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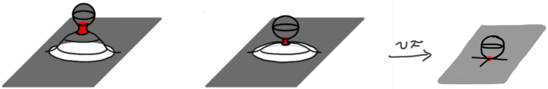

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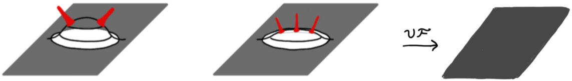

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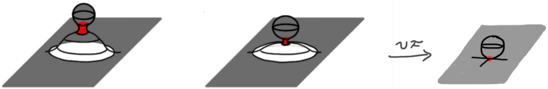

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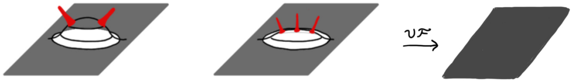

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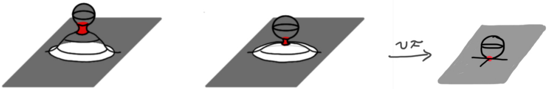

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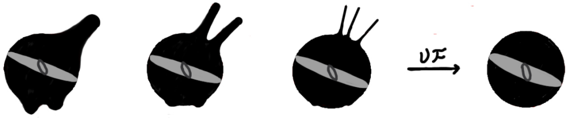

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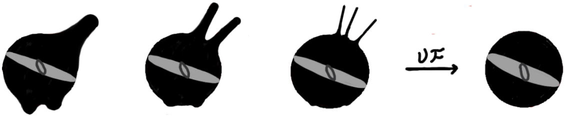

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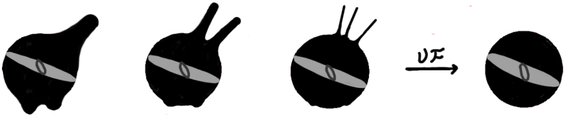

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Hints by Marques-Neves are in [Conj on Convergence and Scalar]

More Geometric Stability Conjectures

In addition to the Geometric Stability of

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See [Conjectures on Conv and Scalar (2021) arXiv: 2103.10093]

Intrinsic Flat and Gromov-Hausdorff Convergence

Lecture 1: Geometric Notions of Convergence DONE!

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Lecture 2: Open Problems about Scalar Curvature DONE!

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will be assigned to teams of interested students/postdocs

Lecture 3&4: How to Prove SWIF and $\mathcal{VF}$ Convergence!

[Conjectures on Convergence and Scalar arXiv: 2103.10093]

Other papers on SWIF convergence may be found at:
https://sites.google.com/site/intrinsicflatconvergence/

This talk may be downloaded at my website:
https://sites.google.com/site/professorsormani/

.

Feel free to email me with questions: sormanic@gmail.com

Thank you for listening - Christina Sormani